Algorithmic Trading Model for Manifold Learning in FX

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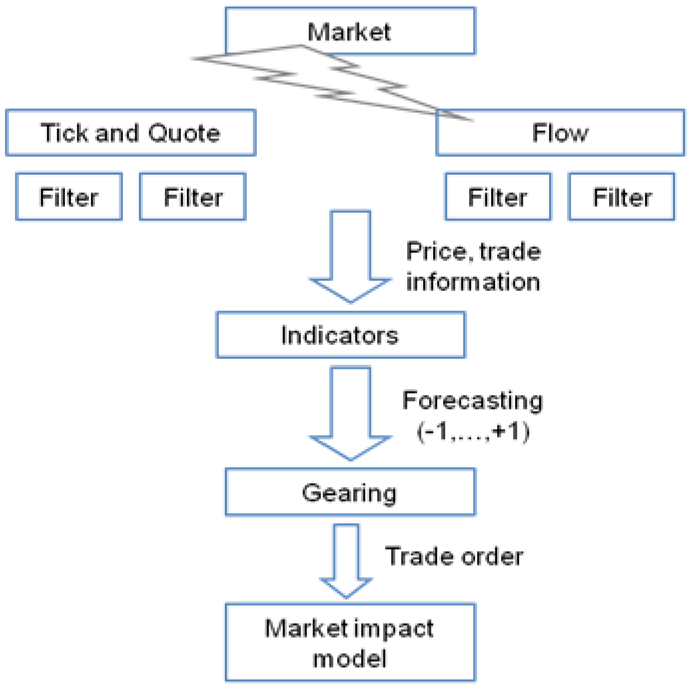
Algorithmic trading ambitions

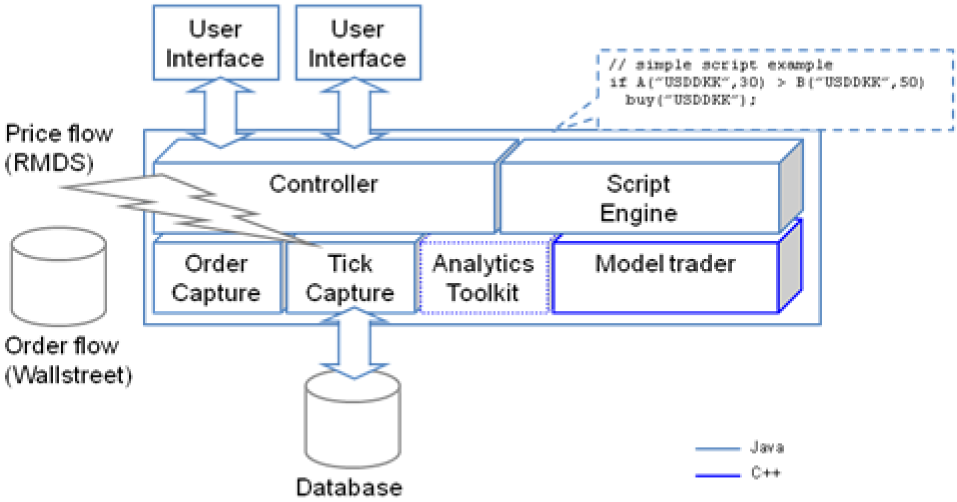
- Facilitate auto-hedging
    - Hold position or sell within a specified timeframe
- Smart price algorithms
    - Skew price within pricing engine
- Proprietary automatic positioning within the foreign exchange market
    - Longer time-scale; combination of manual trading and model implication
    - Shorter timescale; automatic

Building the algorithmic trading system

The system is:

- Connected to
  - Live quotes
  - Proprietary historic quote database
  - Flow data via TradeWarehouse/Wallstreet etc
  - Orderbook history and lost trades archive
  - Making use of the bank's internal analytical toolkit





Time series forecasting

- Traditional timeseries forecasting models of ARMA type

$$
\begin{aligned}
y_t &= c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} \\
&\quad c + \psi_1 v_{t-1} + \psi_2 v_{t-2} + \ldots + \psi_p v_{t-p} \\
&\quad \epsilon_1 + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_p \epsilon_{t-p}
\end{aligned}
$$

- Assumes a set of covariates *v* to govern the response variable *y*
- Standard generalization is to add non-stationary volatility:
  - GARCH assumes serial correlation in volatility

A specific distribution of the returns are assumed

Machine learning approach

- Assume access to many signals *v* that potentially influence *y*
- Straightforward time-series modeling á la GARCH becomes complicated
- Use machine learning methods to automatically extract features of the input signals *v* that carry information about *y*.
- Use these low-dimensional features to predict *y* through regression modelling

Manifold Kernel Dimension Reduction

- A methodology for discovering a data manifold that best preserves information relevant to a nonlinear regression.
- Yields more efficient regression modeling in the lower-dimensional data manifold.

Kernel Dimension Reduction

- Map X and Y reproducing kernel Hilbert spaces $\boldsymbol{H}_X, \boldsymbol{H}_Y$

$$X \mapsto f \in \boldsymbol{H}_X, \quad Y \mapsto g \in \boldsymbol{H}_Y$$

- We can analyze statistical properties of f and g
- Cross-covariance between f and g can be represented by an operator $\boldsymbol{\Sigma}_{YX} : \boldsymbol{H}_X \to \boldsymbol{H}_Y$ such that

$$\langle g, \boldsymbol{\Sigma}_{YX} f \rangle_{\mathcal{H}_{\mathcal{Y}}} = \boldsymbol{C}_{fg}, \quad \forall f, g$$

- Conditional covariance operator

$$\boldsymbol{\Sigma}_{YY|X} = \boldsymbol{\Sigma}_{YY} - \boldsymbol{\Sigma}_{YX}\boldsymbol{\Sigma}_{XX}^{-1}\boldsymbol{\Sigma}_{XY}.$$

KDR Theorem

- Fukumizu, Bach & Jordan (2006):
  -

$$\boldsymbol{\Sigma}_{YY|X} \prec \boldsymbol{\Sigma}_{YY|\boldsymbol{B}^\top X}$$

$$\boldsymbol{\Sigma}_{YY|X} = \boldsymbol{\Sigma}_{YY|\boldsymbol{B}^\top X} \quad \Longleftrightarrow \quad Y \perp X | \boldsymbol{B}^\top X$$

- The central space can be found by minimizing

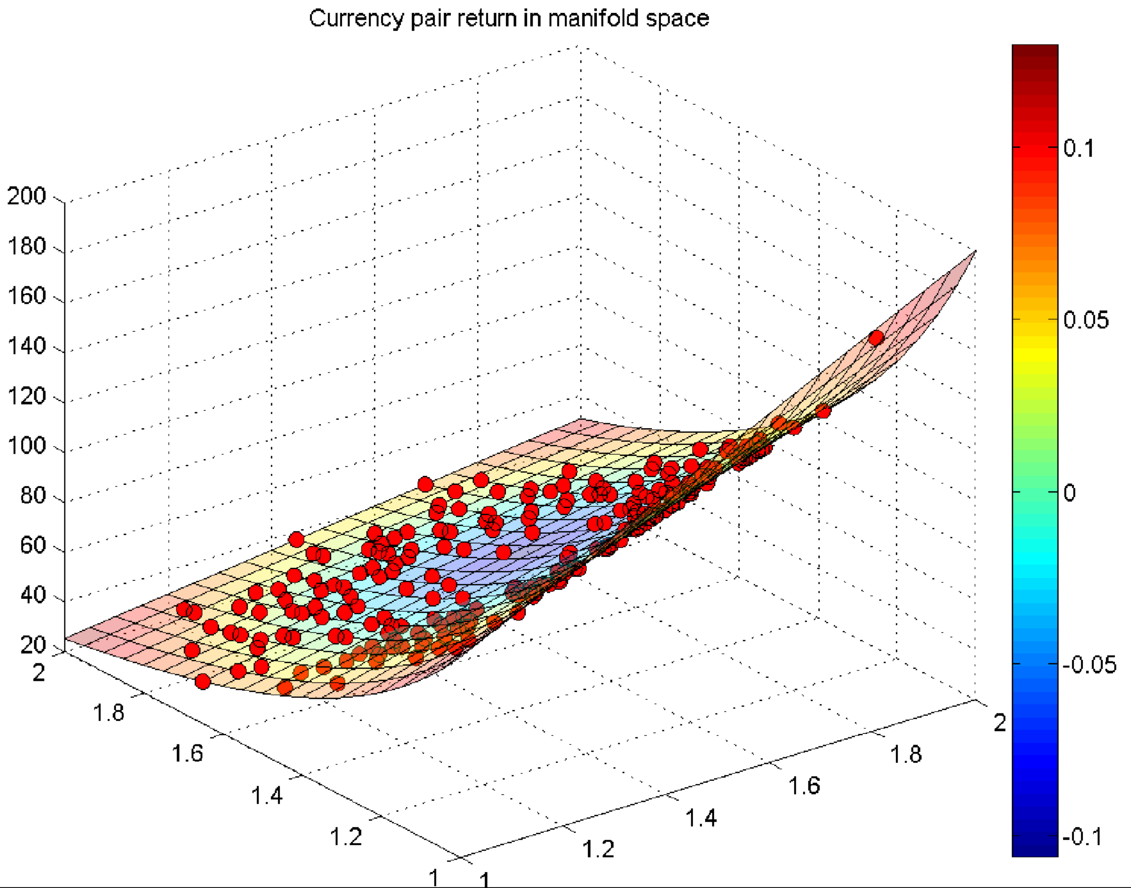
$$\boldsymbol{\Sigma}_{YY|\boldsymbol{B}^\top X}$$

- KDR Algorithm
- The minimization of $\boldsymbol{\Sigma}_{YY|\boldsymbol{B}^\top X}$ can be formulated as

$$\mathrm{Tr}\, || \boldsymbol{K}_Y^c (\boldsymbol{K}_{\boldsymbol{B}^\top X}^c + N\epsilon \boldsymbol{I})^{-1} ||$$

- Min $\boldsymbol{B}^\top \boldsymbol{B} = \boldsymbol{I}$
- such that
- Where $\boldsymbol{K}_Y^c$ and $\boldsymbol{K}_{\boldsymbol{B}^\top X}^c$ are centered kernel matrices.

FX forecasting – Artificial data



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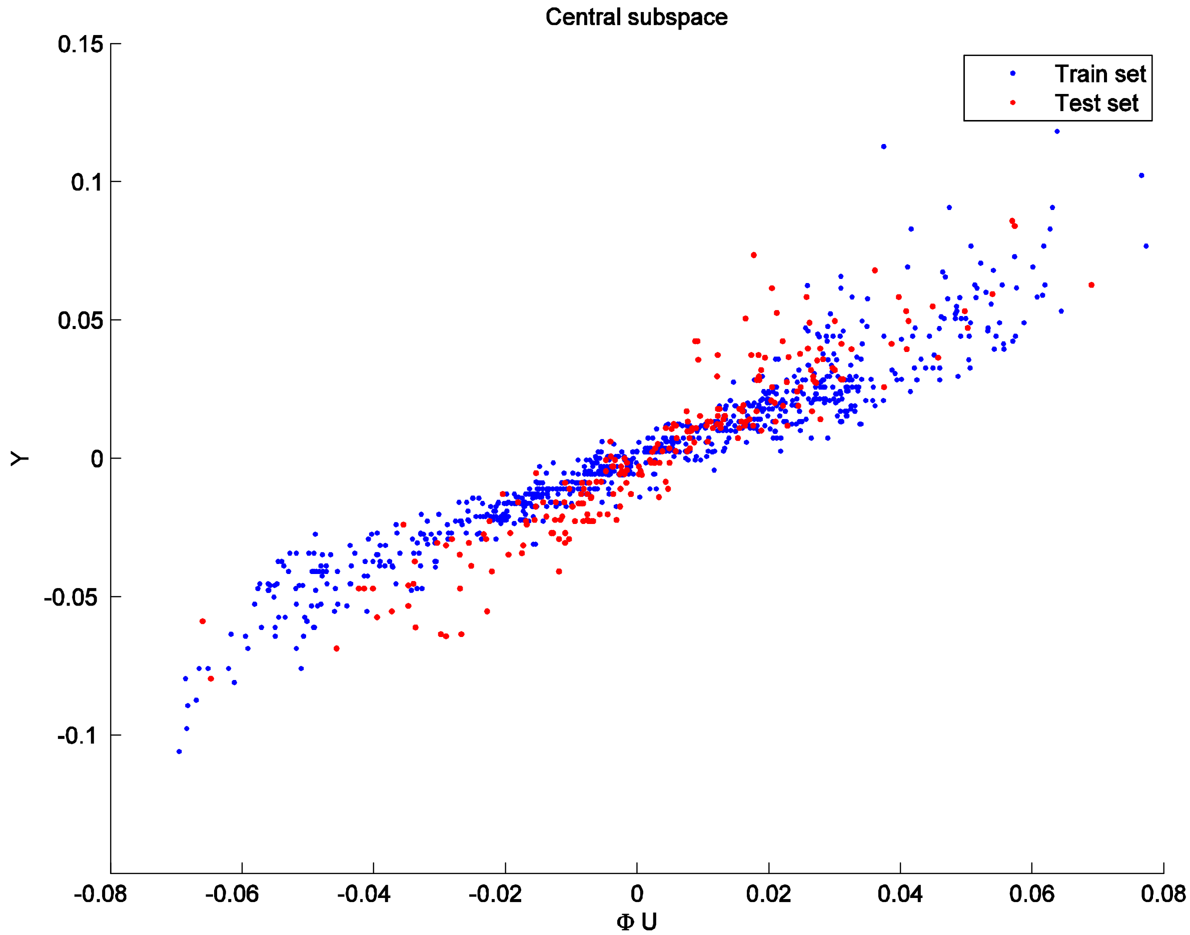
FX forecasting – Artificial data



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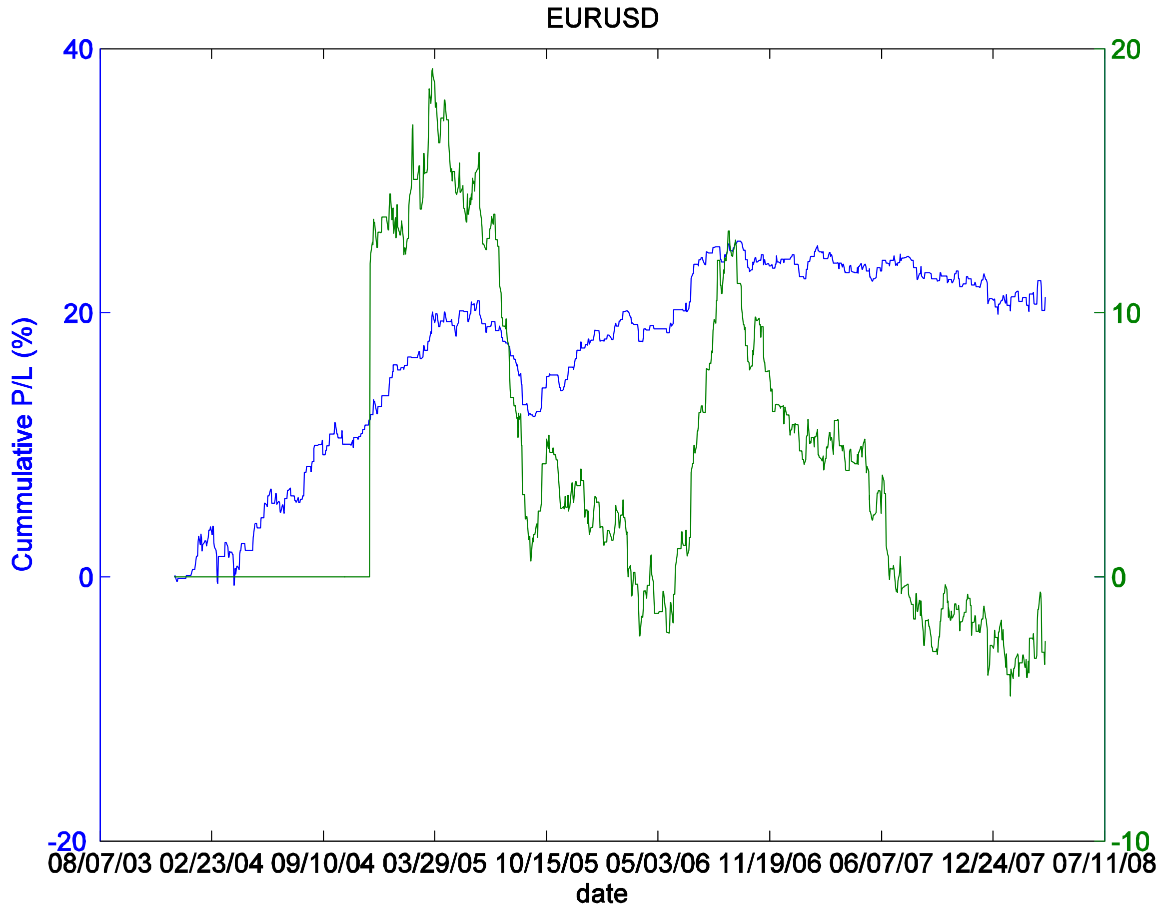
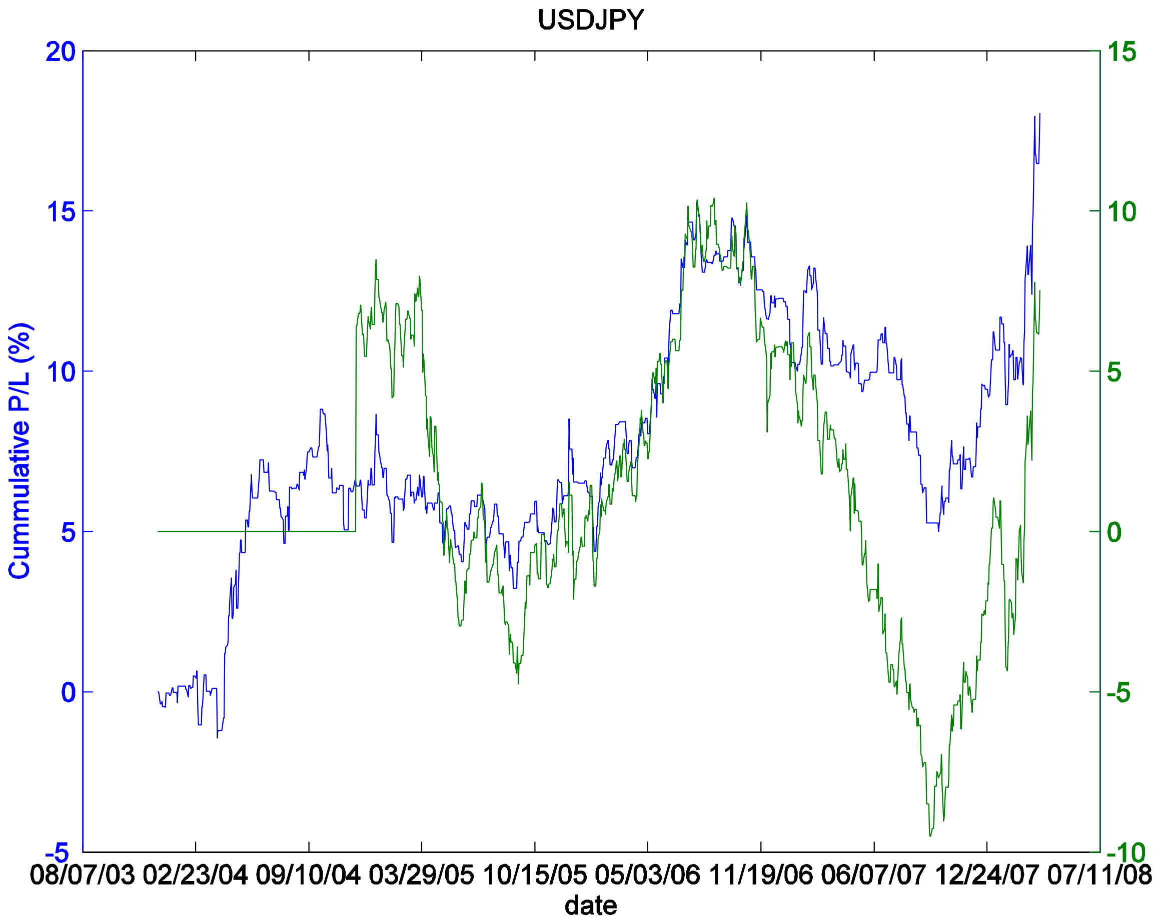
- FX forecasting – Real data
- Simple illustrative example of three major crosses
- EURUSD, USDJPY, EURGBP
- Data for the crosses sampled at daily interval
- Covariates are choosen as the return of the three crosses for one day and one week back

Backtest Results – Real data - EURUSD

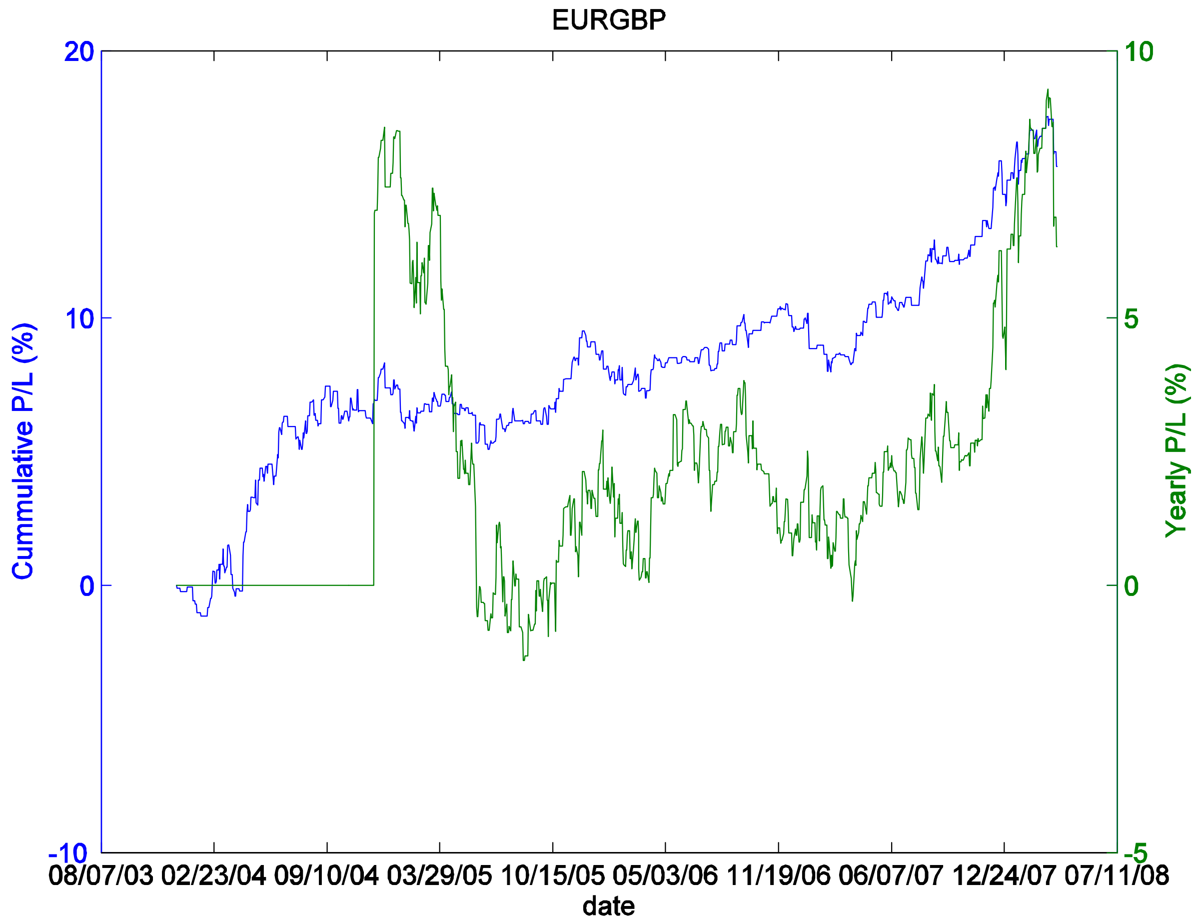
- Positions within each currency pair is taken when the model returns a signal above a given threshold.
- A portfolio consisting of EURUSD, USDJPY, EURGBP is created by weighting the positions according to the signal strengths.



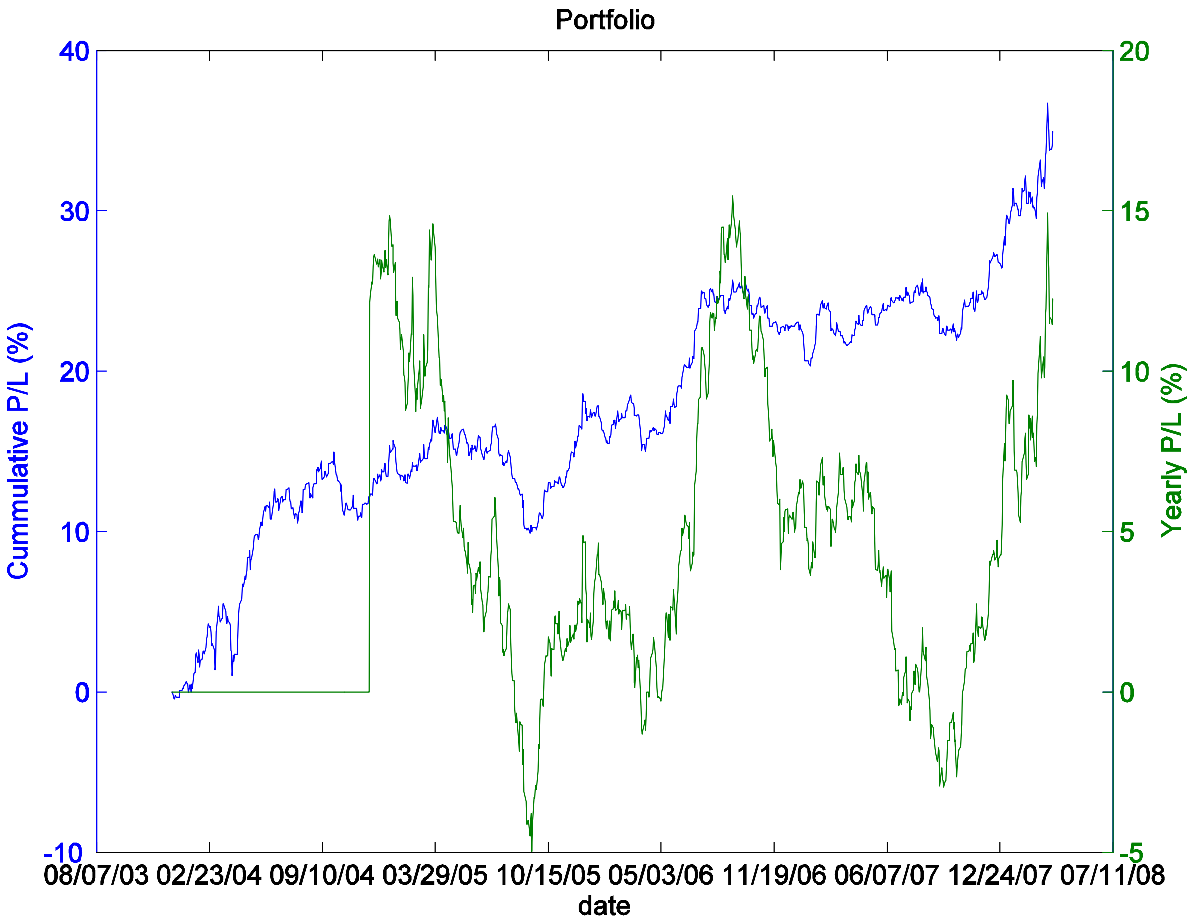
Backtest Results – Real data - USDJPY



Backtest Results – Real data - EURGBP



Backtest Results – Real data - Portfolio



Conclusions

- Prediction of FX movement using manifold learning yields promising results.
- Manifold learning works under general conditions